

The eight Standards for Mathematical Practice will likely give many teachers a reason to pause, celebrate, and possibly double check their teaching practices. No matter what your reaction to these “SMP’s” as the Standards for Mathematical Practice are referred, is, no doubt you’ll be paying attention to them as we inch closer to including these “hows” into our testing culture.

In an effort to help you along this journey of improving your professional practice in regard to the “how’s” of mathematical work, I have compiled the 8 SMP’s in their original language, followed by an easier explanation, and then added the Everyday Math language for each standard at the end of each explanation. The EM language is noted in what we call GMP’s (Goals for Mathematical Practice).

I am not the author of any of this information. I have simply compiled it all here for your convenience. I hope this document is of benefit to you.

If you would like assistance in making sense of these standards and implementing them in your classroom, please reach out. I am here to help you.

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SMP 1 Make sense of problems and persevere in solving them.

Mathematically proficient students **start by explaining** to themselves the meaning of a problem and looking for entry points to its solution. They **analyze** givens, constraints, relationships, and goals. They **make conjectures** about the form and meaning of the solution and **plan a solution** pathway rather than simply jumping into a solution attempt. They **consider** analogous problems, and **try special cases and simpler forms** of the original problem in order to gain insight into its solution. They **monitor and evaluate** their progress and **change course** if necessary. Older students might, depending on the context of the problem, **transform** algebraic expressions or **change** the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can **explain** correspondences between

equations, verbal descriptions, tables, and graphs or **draw** diagrams of important features and relationships, **graph** data, and **search** for regularity or trends. Younger students might rely on **using** concrete objects or pictures to help **conceptualize and solve** a problem. Mathematically proficient students **check** their answers to problems using a different method, and they **continually ask** themselves, “Does this make sense?” They can **understand** the approaches of others to solving complex problems and **identify** correspondences between different approaches.

SMP 1 in bullet points

- Interpret the meaning of a problem.
- Look for multiple ways to approach a problem and develop a plan to solve.
- Monitor and evaluate their progress and change their approach if necessary.
- See relationships between various representations such as concrete objects, graphs or tables, verbal explanations or equations, and notice trends.
- Check answers to problems using different methods.
- Continually ask, "Does this make sense?"

SMP 1 in Everyday Math language- Goals for Mathematical Practice

GMP 1.1 Work to make sense of your problem.

GMP 1.2 Make a plan for solving your problem.

GMP 1.3 Try different approaches when your problem is hard.

GMP 1.4 Solve your problem in more than one way.

GMP 1.5 Check whether your solution makes sense.

GMP 1.6 Connect mathematical ideas and representations to one another.

SMP 2 Reason abstractly and quantitatively.

Mathematically proficient students **make sense of quantities and their relationships in problem situations**. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to **decontextualize**—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to **contextualize**, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of **creating a coherent representation** of the problem at hand; **considering** the units involved; **attending** to the meaning of quantities, not just how to compute them; and **knowing** and flexibly using different properties of operations and objects.

SMP 2 in bullet points

- Know and use the properties of operations.
- Make conjectures and reach conclusions that are logical and defensible.
- Justify their thinking with reasonable arguments.
- Determine whether an assertion is true, a generalization is valid, or an answer correct.

SMP 2 in Everyday Math language- Goals for Mathematical Practice

GMP 2.1 Represent problems and situations mathematically with numbers, words, pictures, symbols, gestures, tables, graphs, and concrete objects.

GMP 2.2 Explain the meanings of the numbers, words, pictures, symbols, gestures, tables, graphs, and concrete objects you and others use.

SMP 3 Construct viable arguments and critique the reasoning of others.

Mathematically proficient students **understand and use** stated assumptions, definitions, and previously established results in constructing arguments. They **make conjectures** and **build a logical progression of statements** to explore the truth of their conjectures. They are able to **analyze situations** by breaking them into cases, and can **recognize and use counterexamples**. They **justify** their conclusions, **communicate** them to others, and **respond** to the arguments of others. They **reason inductively** about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to **compare** the effectiveness of two plausible arguments, **distinguish** correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—**explain** what it is. Elementary students can **construct arguments** using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to **determine domains** to which an argument applies. Students at all grades can **listen or read** the arguments of others, **decide** whether they make sense, and **ask** useful questions to **clarify or improve** the arguments.

SMP 3 in bullet points

- Understand and use stated mathematical assumptions, definitions, and established results in constructing arguments.
- Listen to arguments of others to determine if they make sense.
- Compare two arguments and determine correct or flawed logic.
- Ask useful questions to clarify or improve arguments.

SMP 3 in Everyday Math language- Goals for Mathematical Practice

GMP 3.1 Explain both what to do and why it works.

GMP 3.2 Work to make sense of others' mathematical thinking.

SMP 4 Model with Mathematics

Mathematically proficient students can **apply** the mathematics they know to **solve problems** arising in everyday life, society, and the workplace. In early grades, this might be as simple as **writing** an addition equation to describe a situation. In middle grades, a student might **apply proportional reasoning** to plan a school event or analyze a problem in the community. By high school, a student might **use geometry to solve** a design problem or **use a function to describe** how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable **making assumptions and approximations** to simplify a complicated situation, realizing that these may need revision later. They are able to **identify** important quantities in a practical situation and **map their relationships** using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can **analyze** those relationships mathematically to **draw conclusions**. They routinely **interpret** their mathematical results in the context of the situation and **reflect on** whether the results make sense, possibly improving the model if it has not served its purpose.

SMP 4 in bullet points

- Apply the math they know to solve problems in everyday life.
- Make assumptions or approximations to simplify situations and later make revisions.
- Analyze information in diagrams, flow charts, graphs, etc. and draw conclusions.
- Correctly interpret the results of a mathematical situation.
- Reflect on the results to determine if it makes sense.

SMP 4 in Everyday Math language- Goals for Mathematical Practice

GMP 4.1 Apply mathematical ideas to real world situations.

GMP 4.2 Use mathematical models such as graphs, drawings, tables, symbols, numbers, and diagrams to solve problems.

SMP 5 Use Appropriate Tools Strategically

Mathematically proficient students **consider** the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students **are sufficiently familiar** with tools appropriate for their grade or course to **make sound decisions** about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are **able to identify** relevant external mathematical resources, such as digital content located on a website, and **use them** to pose or solve problems. They are able to **use technological tools** to explore and deepen their understanding of concepts.

SMP 5 in bullet points

- Consider the available tools when solving mathematics problems.
- Make decisions about when available tools might be helpful in solving problems.
- Detect possible errors by strategically using estimation and other mathematical knowledge.
- Identify relevant external mathematical resources to pose and solve problems.
- Use technological tools to explore and deepen understanding of concepts.

SMP 5 in Everyday Math language- Goals for Mathematical Practice

GMP 5.1 Choose appropriate tools for your problem.

GMP 5.2 Use mathematical tools correctly and efficiently.

GMP 5.3 Estimate and use what you know to check the answers you find using tools.

SMP 6 Attend to Precision

Mathematically proficient students try to **communicate precisely** to others. They try to **use clear definitions** in discussion with others and in their own reasoning.

They **state the meaning** of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about **specifying units of measure**, and **labeling axes** to clarify the correspondence with quantities in a problem.

They **calculate** accurately and efficiently, **express** numerical answers with a degree of precision **appropriate for the problem context**. In the elementary grades, students **give carefully formulated** explanations to each other. By the time they reach high school they have learned to **examine claims** and **make explicit use** of definitions

SMP 6 in bullet points

- Communicate precisely to others and give carefully formulated explanations.
- Use symbols and definitions appropriately.
- Carefully label corresponding quantities in problems.
- Calculate accurately and efficiently.

SMP 6 in Everyday Math language- Goals for Mathematical Practice

GMP 6.1 Communicate your mathematical thinking clearly and precisely.

GMP 6.2 Use the level of precision you need for your problem.

GMP 6.3 Be accurate when you count, measure, and calculate.

SMP 7 Look for and Make Use of Structure

Mathematically proficient students look closely to **discern a pattern or structure**. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They **recognize the significance** of an existing line in a geometric figure and can use the **strategy of drawing** an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can **see complicated things**, such as some algebraic expressions, **as single objects** or as **being composed of several objects**. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

SMP 7 in bullet points

- Look closely to discern a pattern or structure.
- They also can step back for an overview and shift perspective.
- They can see complicated things as single objects or as being composed of several objects.

SMP 7 in Everyday Math language- Goals for Mathematical Practice

GMP 7.1 Find, extend, analyze, and create patterns.

GMP 7.2 Use patterns and structures to solve problems.

SMP 8 Look for and express regularity in repeated reasoning

Mathematically proficient students **notice if calculations are repeated**, and **look both for general methods and for shortcuts**. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students **maintain oversight of the process**, while **attending to the details**. They continually **evaluate the reasonableness** of their intermediate results.

SMP 8 in bullet points

- Notice if calculations are repeated, and look both for general methods and for shortcuts. (ex: Upper elementary students might notice, when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal.)
- As they work to solve a problem, they maintain oversight of the process, while attending to the details.
- Continually evaluate the reasonableness of their intermediate results.

SMP 8 in Everyday Math language- Goals for Mathematical Practice

GMP 8.1 Use patterns and structures to create and explain rules and shortcuts.

GMP 8.2 Use properties, rules, and shortcuts to solve problems.

GMP 8.3 Reflect on your thinking before, during, and after solving a problem.